Mapping of the resistivity, susceptibility, and permittivity of the earth using a helicopter-borne electromagnetic system

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ABSTRACT

Interpretation of helicopter-borne electromagnetic (EM) data is commonly based on the mapping of resistivity (or conductivity) under the assumption that the magnetic permeability is that of free space and dielectric permittivity can be ignored. However, the data obtained from a multifrequency EM system may contain information about the magnetic permeability and dielectric permittivity as well as the conductivity. Our previous work has shown how helicopter EM data may be transformed to yield the resistivity and magnetic permeability or, alternatively, the resistivity and dielectric permittivity. A method has now been developed to recover the resistivity, magnetic permeability, and dielectric permittivity together from the transformation of helicopter EM data based on a half-space model. A field example is presented from an area which exhibits both permeable and dielectric properties. This example shows that the mapping of resistivity, magnetic permeability, and dielectric permittivity together yields more credible results than if the permeability or permittivity is ignored.

INTRODUCTION

The three main constitutive equations which relate the behavior of the electromagnetic field to the electrical properties of the earth are (Keller, 1988)

\[ \mathbf{J} = \sigma \mathbf{E}, \]  

\[ \mathbf{D} = \varepsilon \mathbf{E}, \]  

and

\[ \mathbf{B} = \mu \mathbf{H}. \]  

Equation (1) shows that the electric current density \( \mathbf{J} \) is related to the electric field intensity \( \mathbf{E} \) through the electrical conductivity \( \sigma \). Equation (2) shows that the electric charge displacement \( \mathbf{D} \) is related to the electric field intensity \( \mathbf{E} \) through the dielectric permittivity \( \varepsilon \). Equation (3) relates the magnetic induction \( \mathbf{B} \) to the magnetic field intensity \( \mathbf{H} \) through the magnetic permeability \( \mu \). The three constants of proportionality \( \sigma, \varepsilon, \) and \( \mu \) are the main electrical properties of the medium. The purpose of this paper is to show that these three properties can be recovered from the data of a helicopter electromagnetic (EM) survey.

While resistivity has been mapped for many years, the mapping of permeability and permittivity is relatively new. Resistivity mapping was first introduced on a routine basis with the Dighem helicopter EM system (Fraser, 1978). The technique involves the transformation of the EM data to the conductivity or the resistivity (inverse conductivity) of a half-space model. Of the five available single-frequency half-space algorithms (Fraser, 1990), two are in common usage. These are (Figure 1) the homogeneous half-space model, using EM amplitude and system altitude as input, and the pseudolayer half-space model, using in-phase and quadrature components as input. The upper (pseudo) layer of the pseudolayer half-space model of Figure 1b is merely an artifice to account for the difference between the computed sensor-source height \( h \) and the measured sensor altitude \( a \). The measured sensor altitude is determined from the radar or laser altimeter. Any error in the altimeter reading (e.g., caused by a forest canopy) impacts the computed thickness of the pseudolayer but does not corrupt the computed resistivity. The pseudolayer half-space is the model of choice for displaying the apparent resistivity in both plan (Fraser, 1990) and section (Sengpiel, 1988; Huang and Fraser, 1996), in part because of this immunity to altimeter errors.

For forward solutions, the two half-space models are identical since the pseudolayer thickness \( r \) in Figure 1b is set to zero. For inverse and transform solutions, the models differ since
the homogeneous half-space of Figure 1a uses the altitude as input to compute the apparent resistivity, whereas the pseudolayer model of Figure 1b does not. However, both models yield identical values for the computed resistivity if the earth is a true homogeneous half-space.

The computation of resistivity commonly assumes that the magnetic permeability is that of free space and the dielectric permittivity may be ignored. As shown in Figure 2a, this may be envisaged as 1-D parameter-space EM mapping along the β-axis, which is related to conductivity. The resistivity mapping methods work well in areas which are free of magnetic and dielectric polarization. However, they may yield erroneous resistivities in highly magnetic and/or dielectric areas because the in-phase and quadrature responses are affected by magnetic and/or dielectric polarization. With the broad frequency range of some commercial helicopter-borne systems, it is possible to transform the EM data to all three of these electrical parameters.

Huang and Fraser (1998, 2000) developed magnetic permeability and resistivity mapping techniques under the assumption that the earth is a magnetic conductive half-space and that the displacement currents caused by dielectric permittivity could be ignored. This is 2-D parameter-space EM mapping in the conductive-magnetic β-γ plane of Figure 2a, where γ is related to magnetic permeability. Huang et al. (1998) and Huang and Fraser (work in progress) developed a resistivity and dielectric permittivity mapping technique under the assumption that the earth is a dielectric conductive half-space with a magnetic permeability of free space. This is 2-D parameter-space EM mapping in the dielectric-conductive α-β plane, where α is related to dielectric permittivity. Since the conductivity of the earth always affects the EM data in the frequency range...
used in helicopter EM systems, no attempt has been made to perform 2-D mapping in the dielectric-magnetic $\alpha$-$\gamma$ plane.

The effect of dielectric permittivity on the EM data is similar to that of magnetic permeability in that both tend to decrease the in-phase $I$ and increase the quadrature $Q$ responses. For the pseudolayer half-space model, this always results in an increase in the computed resistivity. For the homogeneous half-space model, the computed resistivity may increase or decrease depending on whether the EM amplitude ($=\sqrt{I^2+Q^2}$) increases or decreases.

In resistive areas having magnetic rocks, the magnetic and dielectric effects will both generally be present in high-frequency EM data while only the magnetic effect will exist in low-frequency data. When 2-D parameter-space EM mapping in the dielectric-conductive $\alpha$-$\beta$ plane of Figure 2a is performed, the dielectric permittivity and resistivity computed from the high-frequency data will both be erroneously high if the magnetic permeability is significant. When 2-D parameter-space EM mapping in the conductive-magnetic $\beta$-$\gamma$ plane is performed, the resistivity will tend to be erroneously low if the dielectric permittivity is significant, but the magnetic permeability will be unaffected because it is obtained from the EM data at the lowest frequency. These various problems can be avoided by transforming the EM data to the three electrical parameters of conductivity, permeability, and permittivity.

This paper is concerned with the mapping of any point $p$ in the 3-D parameter-space $\alpha$-$\beta$-$\gamma$ of Figure 2a, using multifrequency helicopter EM data. We provide both synthetic and field examples to show the impact of the magnetic permeability and dielectric permittivity on the computed resistivity. The technique is applicable to any closely-coupled airborne EM system. However, we specifically employ the highest frequencies of Dighem systems, respectively, the Dighem coaxial-coil/coplanar-coil system and the Dighem coaxial-coil/coplanar-coil resistivity mapping system. These commercial helicopter EM systems typically have 380 or 900 Hz as their lowest frequency. In passing, we note that the minimum and maximum frequencies of Dighem systems, respectively, are 220 Hz (for the system owned by Nippon Engineering of Japan) and 195 000 Hz (for the system owned by the German Geological Survey). The latter explains our interest in frequencies as high as 200 000 Hz as used in some of the synthetic examples below.

**IMPACT OF THE ELECTRICAL PARAMETERS ON THE EM RESPONSES**

The effects of conduction, magnetization, and dielectric polarization on the EM data can be found from the forward modeling of half-spaces. We assume that the transmitting and receiving coils are closely coupled, as occur in frequency-domain helicopter EM systems. The ratio of secondary magnetic field intensity $H_s$ to primary magnetic field intensity $H_0$, at the receiving coil, can then be approximated as (Fraser, 1972)

$$H_s/H_0 = (s/h)^3[M(\theta, \mu_r) + iN(\theta, \mu_r)],$$  \hspace{1cm} (4)

where $h$ is the EM sensor-source height (Figure 1b), $s$ is the transmitting-receiving coil separation, and $M$ and $N$ are discussed below. The dimensionless complex induction number $\theta$ for the half-space may be defined as $(\omega^2\varepsilon \mu^2 - i\sigma \omega \mu)^{1/2}$, where $\omega$ is the angular frequency, $\sigma$ is the conductivity, $\mu$ the magnetic permeability, and $\varepsilon$ the dielectric permittivity. In applied geophysics, we often deal with resistivity $\rho = 1/\sigma$, relative magnetic permeability $\mu_r = \mu/\mu_o$, and relative dielectric permittivity (dielectric constant) $\varepsilon_r = \varepsilon/\varepsilon_o$, where $\mu_o$ and $\varepsilon_o$ are the magnetic permeability and dielectric permittivity of free space, respectively. The measured in-phase $I$ and quadrature $Q$ components of the EM response may be represented as

$$I = (s/h)^3 M, \quad Q = (s/h)^3 N,$$  \hspace{1cm} (5)

where the $I$ and $Q$ amplitudes are expressed in units of parts per million (ppm) of the primary magnetic field intensity $H_o$ at the receiving coil. $M$ and $N$ respectively may be referred to as the normalized in-phase and quadrature components because they are simply the measured components $I$ and $Q$ normalized for variations in sensor-source height $h$ and coil separation $s$.

$M$ and $N$ respectively are the in-phase and quadrature components of the complex response function $M + iN$, and are themselves functions of the induction number $\theta$ and the relative magnetic permeability $\mu_r$. Equations (4) and (5) are valid on the assumption that $s \ll h$. This superimposed dipole assumption (Grant and West, 1965) is generally valid for surveys flown with all commercial frequency-domain helicopter EM systems.

For 3-D parameter-space mapping, we employ the orthogonal axes of $\alpha$ and $\beta$ of Figure 2 rather than $s_r$ and $\sigma$ directly. The parameters $\alpha = \omega^2 \varepsilon \mu^2$ and $\beta = \omega \sigma \mu^2$ are, respectively, the real and imaginary parts of the squared induction number $\theta = \omega^2 \varepsilon \mu^2 - i\sigma \omega \mu^2$. The $\gamma$-axis is simply the relative magnetic permeability $\mu_r$.

It is interesting to view the gross behavior of the normalized in-phase $M$ and quadrature $N$ components as the conductivity, magnetic permeability, and dielectric permittivity vary. For example:

1) For the conductive-magnetic $\beta$-$\gamma$ plane, we see that magnetic permeability and conductivity are roughly equal in their impact on the normalized in-phase $M$ component since the colors trend roughly $\pm \frac{\alpha}{\beta}$ to the axes (Figure 2b). However, magnetic permeability has little impact on the normalized quadrature $N$ component (Figure 2c), which is governed primarily by the conductivity, since the colors trend parallel to the $\gamma$-axis.

2) For the dielectric-conductive $\alpha$-$\beta$ plane, the dielectric permittivity has a small impact on the normalized in-phase $M$ (Figure 2b) and very little impact on the quadrature $N$ (Figure 2c). The conductivity has a major impact on both the in-phase and quadrature components.

3) For the dielectric-magnetic $\alpha$-$\gamma$ plane, the dielectric permittivity has a lesser impact on the normalized in-phase $M$ than does the magnetic permeability (Figure 2b), whereas the reverse is true for the quadrature $N$ (Figure 2c).

It is the interaction of the three parameters which is important. All must be dealt with in the transformation of the measured EM responses to allow the correct recovery of the electrical properties of a dielectric magnetic conductive earth.

We may use other presentations to display the in-phase and quadrature components in 3-D parameter space. For example, Figure 3 presents the normalized (a) in-phase $M$ and (b) quadrature $N$ as a function of $\alpha$ and $\beta$ and for three slices...
through the $\gamma = \mu_r$ axis. Each such presentation provides insight into the behavior of the EM response to the three earth parameters.

While the above figures show the gross behavior of the EM, it may help our understanding to view the behavior in detail. Figure 4 shows the measured (a) in-phase $I$ and (b) quadrature $Q$ responses as a function of frequency for four half-space models with variable permeability, and permittivity, but constant resistivity. The impact of the electrical parameters in Figure 4 is as follows.

**Magnetic permeability.**—In comparing model 2 ($\mu_r = 1.02$, $\varepsilon_r = 1$) with model 1 ($\mu_r = \varepsilon_r = 1$), we see that the magnetic permeability $\mu$ causes the measured in-phase $I$ (Figure 4a) to shift down to about $-45$ ppm at the lower frequencies. This in-phase shift caused by magnetic permeability is relatively independent of frequency, as Beard and Nyquist (1998) have shown. When the frequency exceeds 10000 Hz, the in-phase response of both models rises, reaching more than 40 ppm at frequency of 200000 Hz. This rise in the in-phase response is the effect of the conductivity.

**Dielectric permittivity.**—In comparing model 3 ($\mu_r = 1$, $\varepsilon_r = 8$) with model 1 ($\mu_r = \varepsilon_r = 1$), the dielectric permittivity causes the in-phase response of Figure 4a to decrease at the higher frequencies, becoming $-17$ ppm at 200000 Hz. The displacement currents overwhelm the conduction currents for this resistivity of 10000 ohm-m.

**Permeability and permittivity.**—Model 4 encompasses both the magnetic permeability of model 2 and the dielectric permittivity of model 3. The negative in-phase shift (Figure 4a) caused by the magnetic permeability remains at about $-45$ ppm regardless of the dielectric effect at the higher frequencies. In practice, the EM response of magnetic permeability is relatively independent of the dielectric permittivity and the conductivity, while the EM response of dielectric permittivity and conductivity are interdependent.
Figure 4b shows that the magnetic permeability and dielectric permittivity have less of an impact on the measured quadrature responses compared to the in-phase response for these four cases. Nevertheless, the impact on the quadrature can be significant for some model earths and frequencies, as Huang et al. (1998) and Huang and Fraser (2000) have shown.

The example of Figure 4 shows that 3-D parameter-space EM mapping is desirable in areas where the ground is magnetically and dielectrically polarizable, particularly if the in-phase is to be employed in the data transformation. The transformation algorithm, described below, correctly returns the three electrical parameters when the earth is homogeneous.

**DETERMINATION OF MODEL PARAMETERS**

We wish to determine the conductivity, magnetic permeability, and dielectric permittivity. We are given the in-phase and quadrature components of a suite of frequencies, an estimate of the sensor altitude from the altimeter, and the choice of the two half-space models of Figure 1. We first obtain the magnetic permeability from the EM data at the lowest frequency because the ratio of magnetic response to conductive response is maximized and because displacement currents are negligible. We use the homogeneous half-space model of Figure 1a for this purpose (Huang and Fraser, 1998, 2000). The computed magnetic permeability is then used along with the in-phase and quadrature response at the highest frequency to obtain the relative dielectric permittivity, again using the homogeneous half-space model. The highest frequency is used because the ratio of dielectric response to conductive response is maximized.

The apparent resistivity then may be determined from the measured in-phase and quadrature components for each individual frequency, given the relative magnetic permeability \( \mu_r \) and the relative dielectric permittivity \( \varepsilon_r \). The resistivity can be computed using either the homogeneous half-space model of Figure 1a or the pseudolayer half-space model of Figure 1b. We only use the homogeneous half-space model in this paper to ensure that the resistivity is model-consistent with the permeability and permittivity values.

The three electrical parameters of a dielectric magnetic conductive half-space may be obtained by transformation of the in-phase and quadrature components as demonstrated graphically by Figure 5.

Figure 5a shows the phasor diagram which yields the relative magnetic permeability \( \mu_r \) and \( \beta = \omega \sigma \mu h^2 \) from the normalized in-phase \( M \) and quadrature \( N \) responses. The quantity 30000 has been added to \( M \) simply to facilitate plotting in log space by avoiding negative values for the abscissa. While the diagram is applicable to a homogeneous half-space model with variable resistivity, magnetic permeability, and dielectric permittivity, the permittivity is irrelevant because the phasor diagram is plotted in the 2-D conductive-magnetic \( \beta - \gamma \) plane. This approach is permissible since we employ the lowest frequency where displacement currents are negligible. If we wish to solve for the resistivity \( \rho \) at this lowest frequency by using the homogeneous half-space model of Figure 1a, we simply compute \( \rho = 1/\sigma \) directly from \( \beta = \omega \sigma \mu h^2 \) under the assumption that the bird altitude \( a \) from the altimeter can be used in place of the unknown sensor-source height \( h \). The equivalent operator can be written as

\[
\{ \mu_r, \beta \} = f(M, N) \quad (6)
\]

and

\[
\rho = \omega \mu h^2 / \beta. \quad (7)
\]

The \( \beta \) value serves as an indication of the resolvability of the relative permeability \( \mu_r \). If \( \beta \) is low, the value of \( \mu_r \) from Figure 5a is likely to be reliable. If \( \beta \) is greater than 6, the relative permeability \( \mu_r \) will not be reliable.

![Figure 5](image-url)

**Fig. 5.** (a) The phasor diagram of the normalized in-phase \( M \) and quadrature \( N \) response functions for the half-space model of a magnetic conductive earth for several values of \( \beta = \omega \sigma \mu h^2 \) and relative permeabilities \( \mu_r \). Dielectric permittivity is ignored as this diagram is only used for low-frequency data. (b) The phasor diagram of the in-phase \( M \) and quadrature \( N \) for the half-space model of a dielectric conductive magnetic earth for several values of \( \alpha = \omega \varepsilon \mu h^2 \) and \( \beta = \omega \sigma \mu h^2 \). The relative permeability \( \mu_r \) is fixed at 1.05 for this diagram.
In a similar manner, Figure 5b can be used to yield the dielectric permittivity and resistivity from the normalized in-phase and quadrature data at the highest frequency, given the relative permeability \( \mu_r \). Figure 5b is derived from a 3-D image with axes of \( M, N, \) and \( \mu_r \). For simplicity, only the slice for \( \mu_r = 1.05 \) is displayed. The equivalent operator is

\[
\{\rho, \varepsilon_r\} = f(M, N, \mu_r).
\]

Knowing the magnetic permeability and dielectric permittivity, we are able to determine the apparent resistivity for all frequencies. If the homogeneous half-space model (Figure 1a) is used, the resistivity for the lowest and the highest frequencies can be obtained directly from equations (7) and (8).

The apparent resistivities for all the remaining frequencies may be obtained from the homogeneous half-space model of Figure 1a as

\[
\rho = f(M, N, \varepsilon_r, \mu_r),
\]

where the relative permittivity and permeability are inputs.

On the other hand, if the pseudolayer half-space model (Figure 1b) is used to compute the resistivity, the equivalent algorithm can be written as

\[
\{\rho, h\} = f(I, Q, \varepsilon_r, \mu_r),
\]

where the output sensor-source height \( h \) is defined in Figure 1b and the inputs \( \mu_r \) and \( \varepsilon_r \) are first obtained from equations (6) and (8). The apparent thickness (Figure 1b) then is calculated as

\[
t_a = h_a - a.
\]

If the earth is a true homogeneous half-space, the computed resistivities, the relative magnetic permeability \( \mu_r \), and the relative dielectric permittivity \( \varepsilon_r \) would be the true values, and \( \tau_a \) would be zero. Otherwise, they would be apparent values.

SYNTHETIC DATA EXAMPLES

We present some examples from synthetic data to show the importance of 3-D parameter-space mapping. We first examine two homogeneous half-space cases, one for dielectric permittivity-conductivity mapping in the \( \alpha-\beta \) plane versus 3-D mapping, and the other for conductivity-magnetic permeability mapping in the \( \beta-\gamma \) plane versus 3-D mapping. The third example shows a two-layer model with resistivity, permeability, and permittivity contrasts. The electrical parameters in these examples are consistent with some environments in northern Canada, apart from some of the higher values for the dielectric permittivity.

Figure 6 presents the results from a homogeneous half-space model with a resistivity of 10000 ohm-m and a relative magnetic permeability of 1.05 (representing about 1.5% magnetite). The relative dielectric permittivity varies, increasing along the abscissa from 1 to 100. This model was used to generate the in-phase and quadrature responses for both 56000 and 105 000 Hz. These EM responses were transformed into the apparent resistivity and the apparent dielectric permittivity using the \( \alpha-\beta \) plane 2-D algorithm. Figure 6a shows that the apparent resistivities \( \rho_a \) computed for the two frequencies are significantly overestimated in the dielectric-conductive \( \alpha-\beta \) plane mapping where magnetic permeability is ignored. When the magnetic permeability, as obtained from the low frequency of 900 Hz, is used as an input to the 3-D mapping algorithm, the true resistivity is obtained as shown in the lowermost of the three curves of Figure 6a.

Figure 6b shows that the computed apparent dielectric permittivity \( \varepsilon_{ra} \) is also significantly overestimated in the dielectric-conductive \( \alpha-\beta \) plane mapping. In particular, when the dielectric permittivity of the ground is low, a false dielectric high is generated due to the ignored magnetic permeability. As for the above resistivity computation, when the magnetic permeability obtained from the lowest frequency is used as an input to the 3-D mapping algorithm, the true dielectric permittivity is obtained as seen in the lowermost curve. The errors for both the apparent resistivity and the apparent dielectric permittivity
are smaller for 105 000 Hz than for 56 000 Hz because the EM data at the higher frequency contains a proportionately smaller contribution from the magnetic permeability.

Figure 7 shows the effect on the computed resistivity in the 2-D conductive-magnetic \(\alpha-\beta\) plane mapping where the dielectric permittivity is ignored. The model in Figure 7a is a homogeneous half-space with a relative permeability of 1.05, a dielectric permittivity which ranges discretely from 10 to 80, and a resistivity which increases continuously from 1000 to 100 000 ohm-m along the abscissa. The model in Figure 7b is also a homogeneous half-space but with a fixed resistivity of 20 000 ohm-m, a dielectric permittivity which ranges discretely from 10 to 80, and a relative magnetic permeability which increases continuously from 1 to 2 along the abscissa. For both of these models, the computed EM data were transformed into the apparent resistivity and the apparent magnetic permeability using the conductive-magnetic 2-D algorithm for the \(\beta-\gamma\) plane. This algorithm yielded the true value for the magnetic permeability from the low frequency of 380 Hz. However, the apparent resistivity computed from 105 000 Hz EM data is, in general underestimated in the \(\beta-\gamma\) plane mapping where dielectric permittivity is ignored. The errors in the resistivity increase with the true resistivity and with the dielectric permittivity. When the 3-D parameter-space mapping algorithm is used, the true resistivity is obtained as is shown in the uppermost curve of both parts of Figure 7. The errors in the apparent resistivity are smaller for the frequency of 56 000 Hz (not shown) than for the frequency of 105 000 Hz because the dielectric permittivity has less impact on the lower frequency EM data.

Figure 8a shows a two-layer model with the thickness of the upper layer increasing to the right. The two layers have differing values for the electrical parameters. The computed apparent resistivity \(\rho_a\) (Figure 8b), apparent magnetic susceptibility \(\kappa_a = \mu_a - 1\) (Figure 8c), and apparent dielectric permittivity \(\varepsilon_{ra}\) (Figure 8d) are shown as functions of the upper-layer thickness \(t_1\). The apparent relative magnetic permeability was first determined for each thickness from low-frequency EM data and then recast as susceptibility (for display purposes) as shown in Figure 8c. The apparent permeability was then used along with the EM responses at 105 000 Hz to obtain the apparent resistivity and apparent dielectric permittivity for each thickness using both the 2-D and 3-D algorithms. The resistivity and dielectric permittivity are overestimated in the 2-D dielectric-conductive \(\alpha-\beta\) plane mapping shown in Figures 8b and 8d. The resistivity is underestimated in the 2-D conductive-magnetic \(\beta-\gamma\) plane mapping in Figure 8b. The results from the 3-D mapping in \(\alpha-\beta-\gamma\) space are most consistent with the model since the apparent values on the left side of Figures 8b and 8d (where the upper layer is thinnest) are closest to the true parameters of the lower layer.

**A FIELD EXAMPLE**

Field examples tend to encompass many resistivity, permeability, and permittivity variations in a single survey. They can at times provide a qualitative view of the usefulness of an analytic method. We will present an example of field data from northern Canada showing the crosscoupling of the computed electrical properties when a parameter is ignored. The highest and lowest frequencies of the Dighem system used in this survey are 56 000 Hz and 900 Hz respectively.

The apparent magnetic permeability \(\mu_a\) is derived from the lowest frequency data and then converted to the apparent magnetic susceptibility \(\kappa_a = \mu_a - 1\), as shown in Figure 9b. The similarity with the magnetometer map of Figure 9a is to be expected since the geology is steeply dipping and the overburden is thin. Both the EM system and the magnetometer are virtually sampling the same geology even though the depth of exploration of the magnetometer is much greater than that of the helicopter EM system. The EM-derived magnetic susceptibility of Figure 9b displays a higher resolution of the minor features than the total field magnetic map of Figure 9a. This is evident in the eastern portion of the survey area. The higher resolution for the EM-derived susceptibility is not surprising since the dipole magnetic field of the EM transmitting coil has a higher resolution, and a rapid falloff with distance from the coil, compared to the uniform magnetic field of the earth.

Figures 10a and 10b respectively show the apparent resistivity and the apparent dielectric permittivity computed from the EM data at 56 000 Hz using 2-D dielectric-conductive \(\alpha-\beta\) plane mapping (i.e., magnetic permeability is ignored). Some of the magnetic features on the apparent magnetic susceptibility map of Figure 9b (e.g., the magnetic dike of arrow A) appear as...
artifacts on the maps of apparent resistivity (Figure 10a) and dielectric permittivity (Figure 10b). The magnetic rocks also cause the computed resistivity and dielectric permittivity to be too high since the in-phase response is decreased by magnetic polarization.

We now consider the effect of 2-D mapping in the $\beta$-$\gamma$ plane, which ignores dielectric permittivity. The 56,000 Hz apparent resistivity map of Figure 10c may be compared with the dielectric permittivity map of Figure 10b. It can be seen that the apparent resistivity contains artifacts (e.g., arrow B) caused by the dielectric permittivity. Also, the values of resistivity obtained using the 2-D method (Figure 10c) are, in general, lower than the values of resistivity obtained when using the 3-D method (Figure 11a).

Figures 11a and 11b present the apparent resistivity and apparent dielectric permittivity map derived from the EM data

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**Fig. 8.** Comparison of 2-D plane mapping with the 3-D parameter-space mapping for a two-layer model. (a) The two-layer model where the thickness of the upper layer increases from left to right. (b) The apparent resistivity $\rho_a$, (c) apparent magnetic susceptibility $\kappa_a$ and (d) apparent dielectric permittivity $\varepsilon_{ra}$ are shown as functions of upper-layer thickness $t_1$ for the 2-D and 3-D algorithms.

**Fig. 9.** (a) The total magnetic field from a magnetometer. (b) The apparent magnetic susceptibility obtained from the in-phase and quadrature components at 900 Hz (after Huang and Fraser, 2000). Arrow A identifies a feature described in the text.
FIG. 10. (a) The apparent resistivity and (b) the apparent dielectric permittivity obtained from the in-phase and quadrature components at 56,000 Hz using 2-D $\alpha$-$\beta$ plane mapping for the same area as Figure 9. (c) The apparent resistivity obtained from the in-phase and quadrature components at 56,000 Hz using 2-D $\beta$-$\gamma$ plane mapping where the permeability was obtained from the 900-Hz data. Arrows A and B identify features referred to in the text.

FIG. 11. (a) The apparent resistivity and (b) the apparent dielectric permittivity computed from the in-phase and quadrature components at 56,000 Hz using 3-D $\alpha$-$\beta$-$\gamma$ mapping for the same area as Figure 9.
at 56,000 Hz using the 3-D (α-β-γ) mapping method. The magnetic features do not appear as artifacts on the maps of resistivity and dielectric permittivity. Similarly, the dielectric features do not appear as artifacts on the resistivity map. The three electrical parameters of magnetic susceptibility (Figure 9b), resistivity (Figure 11a), and dielectric permittivity (Figure 11b) appear to be satisfactorily decoupled. This illustrates the advantage of separately mapping the resistivity, magnetic permeability, and dielectric permittivity in resistive terrain.

CONCLUSIONS

New methods for the mapping of the three main electrical properties of the earth have been developed for helicopter-borne electromagnetic systems. The model used in this study is a half-space with variable resistivity, magnetic permeability, and dielectric permittivity. Since the EM data obtained from the lowest frequency is typically free of dielectric effect, the relative magnetic permeability can be obtained from the in-phase and quadrature response at this lowest frequency. The computed magnetic permeability is then used along with the in-phase and quadrature response at the highest frequency to obtain the relative dielectric permittivity. The resistivity may be computed for each frequency, using the magnetic permeability and dielectric permittivity as input if required. This procedure avoids errors in the computed resistivity and dielectric permittivity caused by the magnetic effect, and it avoids errors in the computed resistivity from the dielectric effect.

A field example shows that the resistivity, magnetic susceptibility, and dielectric permittivity appear to be satisfactorily decoupled, illustrating the advantage of separately mapping these three main electrical properties of the earth.

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